

# Practical Variable Step-Size Adaptive Algorithms for Echo Cancellation

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# Introduction

- **adaptive echo cancellation**

- an *adaptive filter* identifies the echo path

- resulted due to the hybrid circuit (2 wire  $\leftrightarrow$  4 wire)  
[*network echo cancellation* (NEC)]

- between the terminal's loudspeaker and microphone  
[*acoustic echo cancellation* (AEC)]

- **specific problems in echo cancellation**

- the echo path can be extremely long

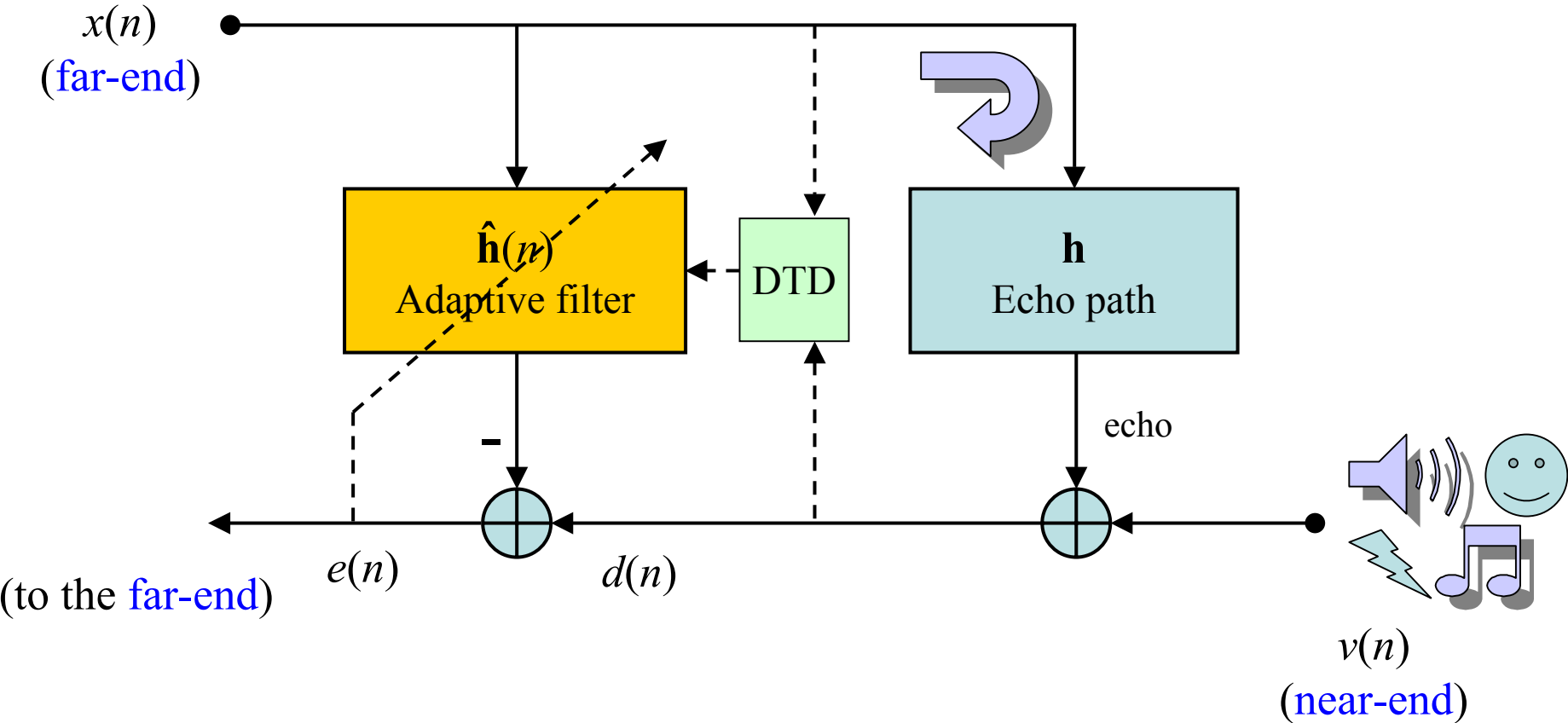
- it may rapidly change at any time during the connection

- the background noise can be strong and non-stationary


- the behaviour during double-talk

- the presence of the Double-Talk Detector (DTD)

# Basic configuration



# Adaptive algorithms for echo cancellation

- **requirements**
    - fast convergence rate and tracking
    - low misadjustment
    - double-talk robustness
  - **common choices**
    - Normalized least-mean-square (NLMS) algorithm
    - Affine Projection Algorithm (APA)
  - **step-size parameter**
    - *large* values → fast convergence rate and tracking
    - *small* values → low misadjustment and double-talk robustness
-  **conflicting requirements → variable step-size (VSS) algorithms**

- **classical NLMS algorithm**

$$e(n) = d(n) - \mathbf{x}^T(n)\hat{\mathbf{h}}(n-1)$$

$$\hat{\mathbf{h}}(n) = \hat{\mathbf{h}}(n-1) + \mu(n)\mathbf{x}(n)e(n)$$

step-size parameter

where  $\mu(n) = \mu / [\mathbf{x}^T(n)\mathbf{x}(n)]$

adaptive filter length

$$\mathbf{x}(n) = [x(n), x(n-1), \dots, x(n-L+1)]^T$$

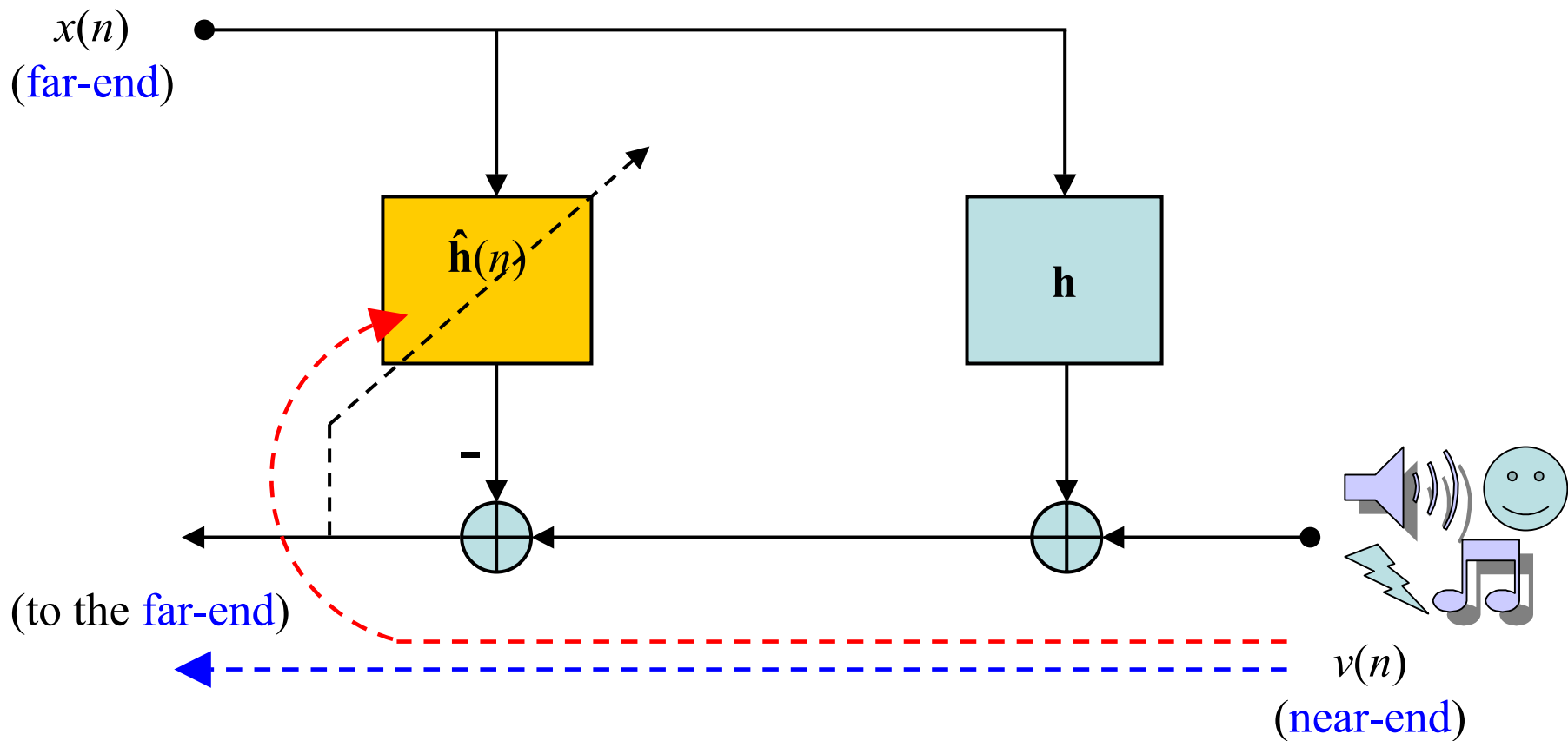
$$\varepsilon(n) = d(n) - \mathbf{x}^T(n)\hat{\mathbf{h}}(n) \rightarrow \text{a posteriori error vector}$$

$$\rightarrow \varepsilon(n) = e(n)[1 - \mu(n)\mathbf{x}^T(n)\mathbf{x}(n)]$$

$$\varepsilon(n) = 0 \quad [\text{assuming that } e(n) \neq 0] \quad \rightarrow \quad \mu(n) = 1 / \mathbf{x}^T(n)\mathbf{x}(n)$$

$$\rightarrow \mu = 1$$

**Optimal step-size ?**



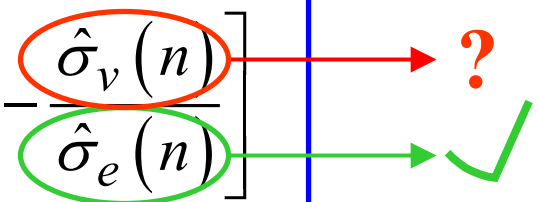
~~$\varepsilon(n) = 0$~~

$\varepsilon(n) = v(n)$

$$E\{\varepsilon^2(n)\} = E\{v^2(n)\}$$

$$E\{e^2(n)\} [1 - \mu(n) L E\{x^2(n)\}]^2 = E\{v^2(n)\}$$

$$\rightarrow \mu(n) = \frac{1}{\mathbf{x}^T(n)\mathbf{x}(n)} \left[ 1 - \sqrt{\frac{E\{v^2(n)\}}{E\{e^2(n)\}}} \right]$$

$$\mu(n) = \frac{1}{\mathbf{x}^T(n)\mathbf{x}(n)} \left[ 1 - \frac{\hat{\sigma}_v(n)}{\hat{\sigma}_e(n)} \right]$$


$$\hat{\sigma}_\alpha^2(n) = \lambda \hat{\sigma}_\alpha^2(n-1) + (1-\lambda) \alpha^2(n) \quad \lambda = 1 - 1/(KL), \text{ with } K > 1$$

1) near-end signal = background noise (*single-talk scenario*)

$$v(n) = w(n)$$

$$\mu(n) = \frac{1}{\mathbf{x}^T(n)\mathbf{x}(n)} \left[ 1 - \frac{\hat{\sigma}_w}{\hat{\sigma}_e(n)} \right]$$

background noise power estimate

[J. Benesty *et al*, "A nonparametric VSS NLMS algorithm", *IEEE Signal Process. Lett.*, 2006]

*Problem:* background noise can be time-variant

2) near-end signal = background noise + near-end speech

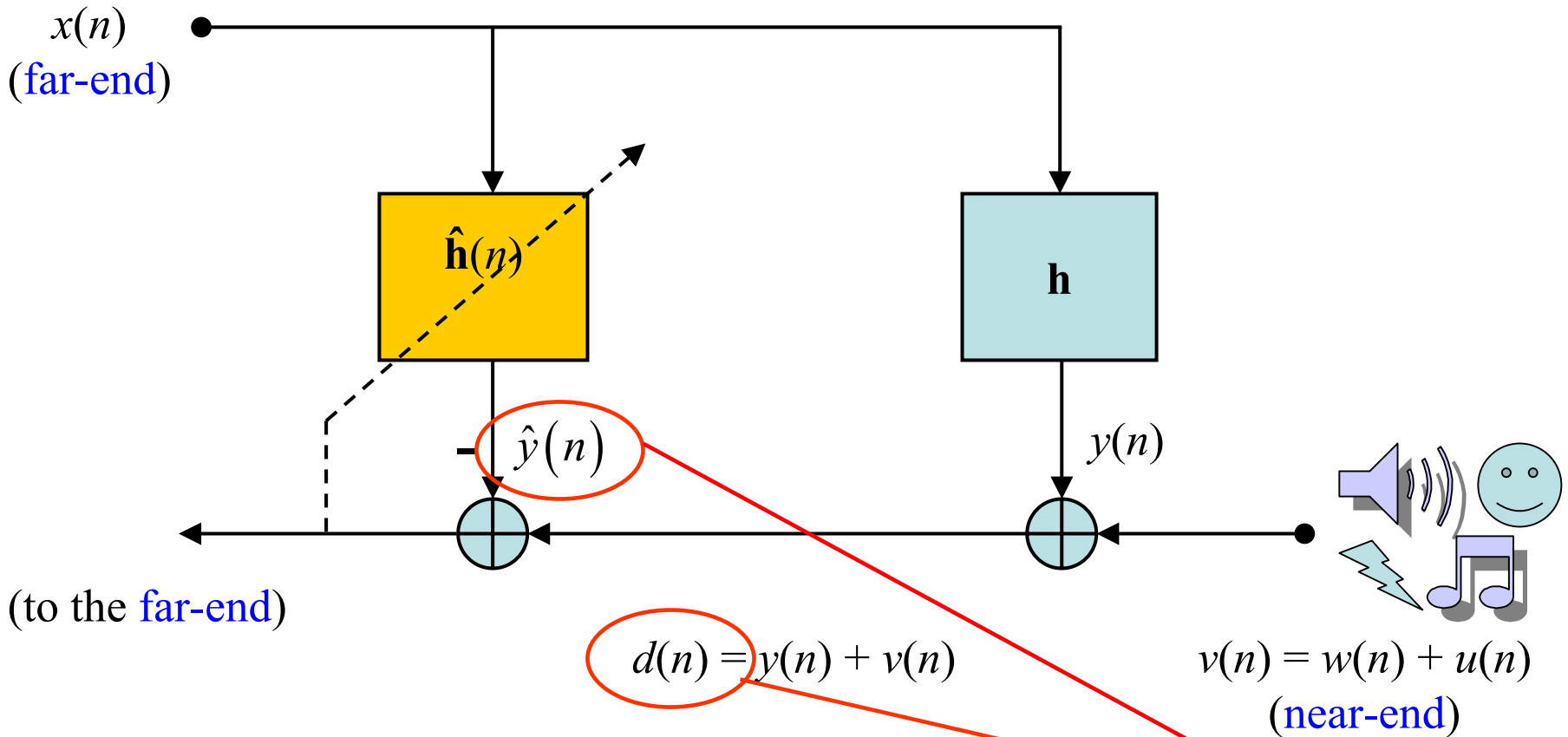
$$v(n) = w(n) + u(n) \quad (\textit{double-talk scenario})$$

$$\hat{\sigma}_v^2(n) = \hat{\sigma}_w^2(n) + \hat{\sigma}_u^2(n)$$

near-end speech power estimate

???

*Problem:* non-stationary character of the speech signal



$$E\{d^2(n)\} = E\{y^2(n)\} + E\{v^2(n)\}$$

$$E\{y^2(n)\} \cong E\{\hat{y}^2(n)\}$$

→ assuming that the adaptive filter has converged to a certain degree

$$E\{v^2(n)\} \cong E\{d^2(n)\} - E\{\hat{y}^2(n)\}$$

$$\hat{\sigma}_v^2(n) \cong \hat{\sigma}_d^2(n) - \hat{\sigma}_{\hat{y}}^2(n)$$

$$\mu(n) = \frac{1}{\mathbf{x}^T(n)\mathbf{x}(n)} \left[ 1 - \frac{\sqrt{\hat{\sigma}_d^2(n) - \hat{\sigma}_{\hat{y}}^2(n)}}{\hat{\sigma}_e(n)} \right]$$

**Proposed  
VSS-NLMS**

- **main advantages**
  - non-parametric algorithm
  - robustness to background noise variations and double-talk
- **the idea can be extended to the APA case → VSS-APA**
  - *superior convergence rate* as compared to the VSS-NLMS algorithm
  - *lower complexity* as compared to the recursive least-squares (RLS) algorithm

- **classical APA**

$$\mathbf{e}(n) = \mathbf{d}(n) - \mathbf{X}^T(n) \hat{\mathbf{h}}(n-1)$$

$$\hat{\mathbf{h}}(n) = \hat{\mathbf{h}}(n-1) + \mu \mathbf{X}(n) \left[ \mathbf{X}^T(n) \mathbf{X}(n) \right]^{-1} \mathbf{e}(n)$$

where  $\mathbf{d}(n) = [d(n), d(n-1), \dots, d(n-p+1)]^T$  ← projection order

$$\mathbf{X}(n) = [\mathbf{x}(n), \mathbf{x}(n-1), \dots, \mathbf{x}(n-p+1)]$$

$$\mathbf{x}(n-l) = [x(n-l), x(n-l-1), \dots, x(n-l-L+1)]^T$$

$$l = 0, 1, \dots, p-1$$

← adaptive filter length

$$\mu_l(n) = 1 - \frac{\sqrt{\hat{\sigma}_d^2(n-l) - \hat{\sigma}_y^2(n-l)}}{\hat{\sigma}_{e_{l+1}}(n)} \quad \text{Proposed VSS-APA}$$

$$\boldsymbol{\mu}(n) = \text{diag} \{ \mu_0(n), \mu_1(n), \dots, \mu_{p-1}(n) \}$$

$$\hat{\mathbf{h}}(n) = \hat{\mathbf{h}}(n-1) + \mathbf{X}(n) \left[ \mathbf{X}^T(n) \mathbf{X}(n) \right]^{-1} \boldsymbol{\mu}(n) \mathbf{e}(n)$$

# Simulation results

- **conditions**

- AEC context,  $L = 512$

- $x(n), u(n)$  = speech sequences

- $w(n)$  = independent white Gaussian noise signal (SNR = 20dB)

- normalized misalignment (dB) =  $20 \log_{10}(\|\mathbf{h} - \hat{\mathbf{h}}(n)\| / \|\mathbf{h}\|)$

- **algorithms for comparisons**

- classical APA,  $\mu = 0.2$

- variable regularized APA (VR-APA)

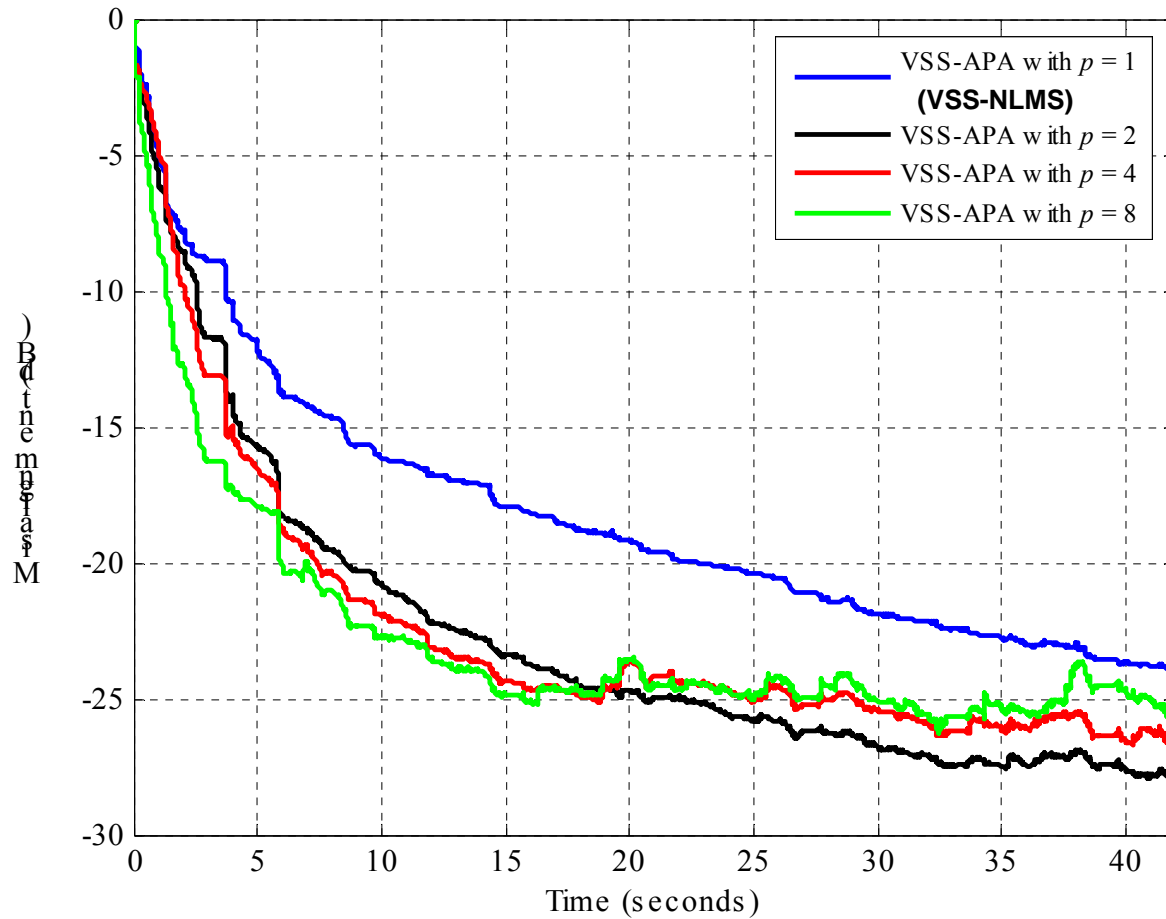
- [H. Rey, L. Rey Vega, S. Tressens, and J. Benesty, *IEEE Trans. Signal Process.*, May 2007]

- robust proportionate APA (R-PAPA)

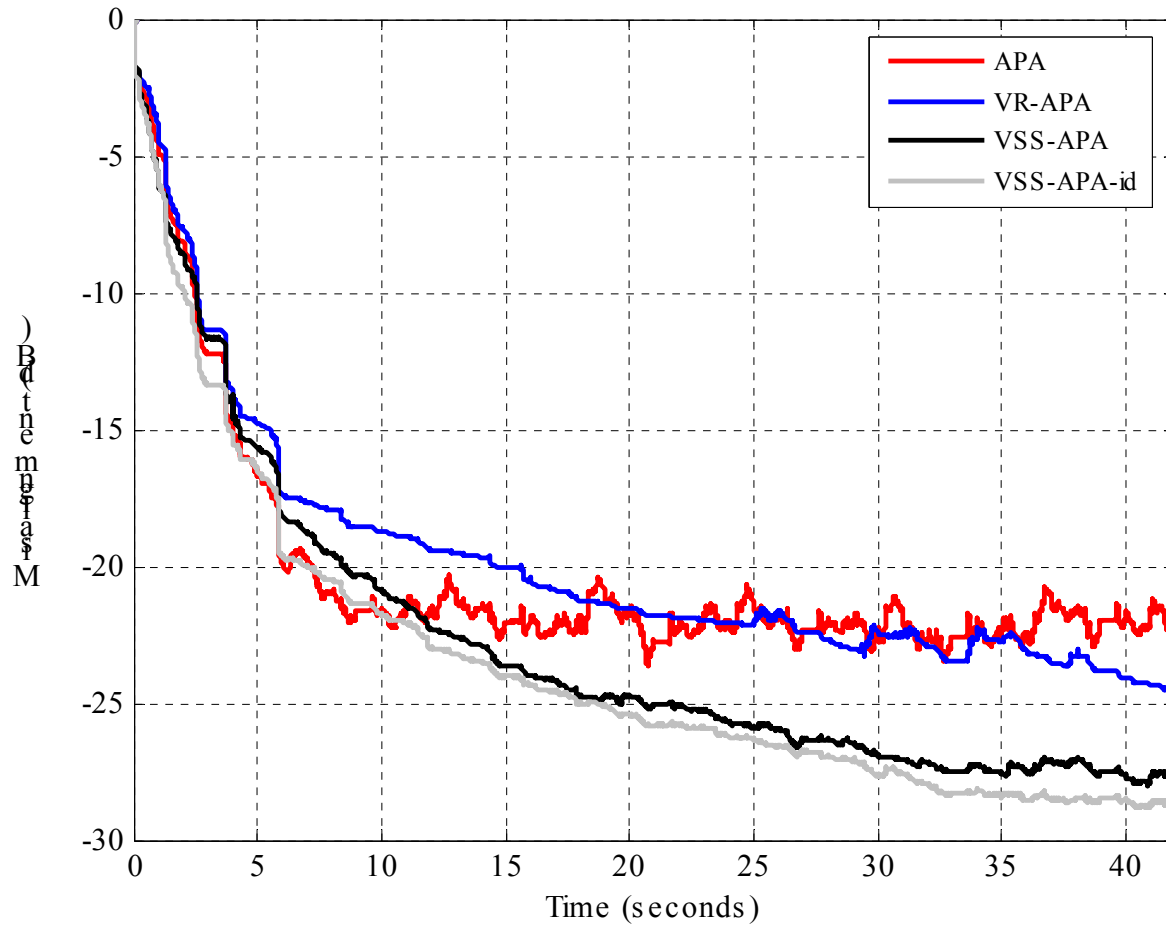
- [T. Gänslér, S. L. Gay, M. M. Sondhi, and J. Benesty, *IEEE Trans. Speech Audio Process.*, Nov. 2000]

- “ideal” VSS-APA (VSS-APA-id) - assuming that  $v(n)$  is available

- single-talk scenario (near-end signal = background noise)

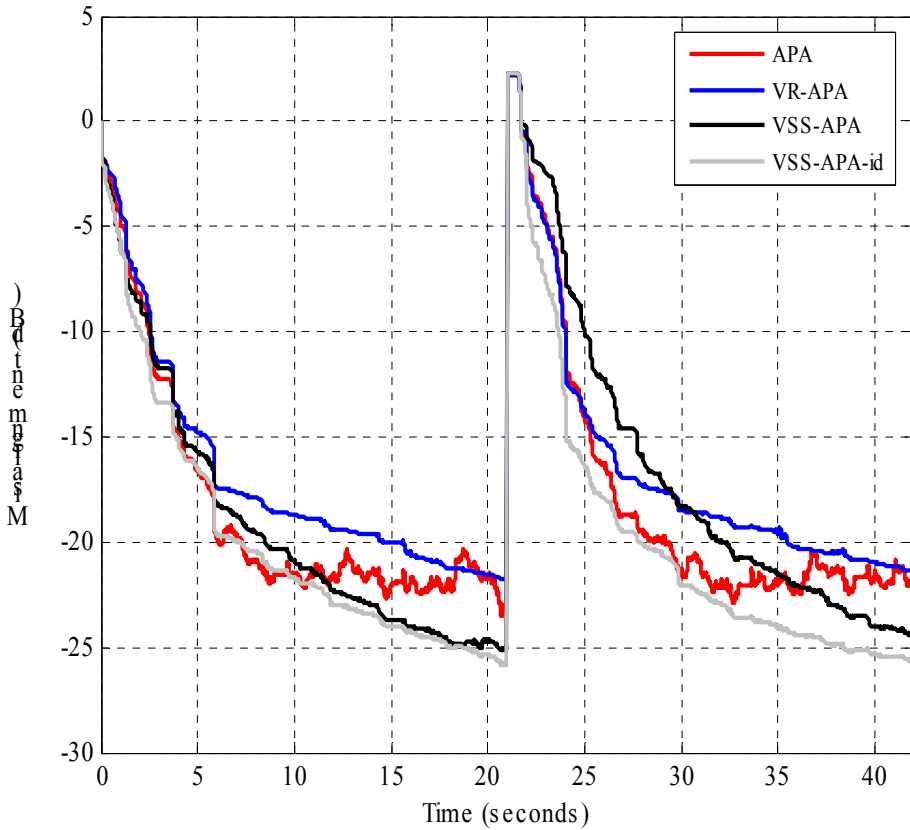


- single-talk scenario (near-end signal = background noise),  $p = 2$

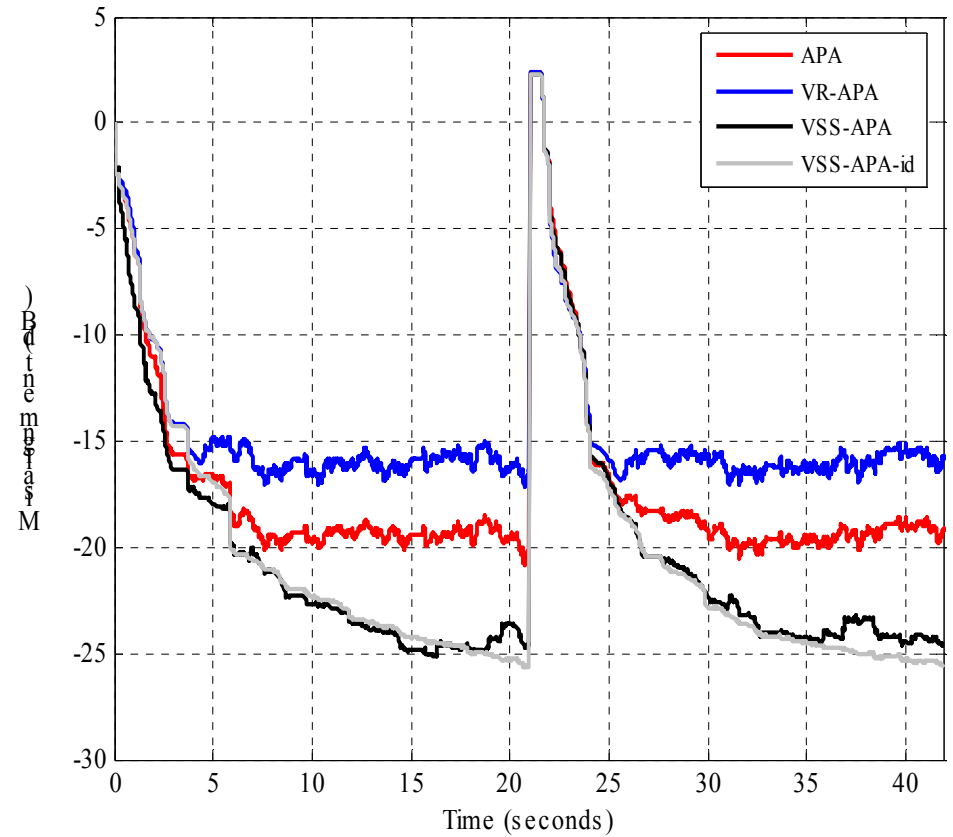


- single-talk scenario + echo path changes

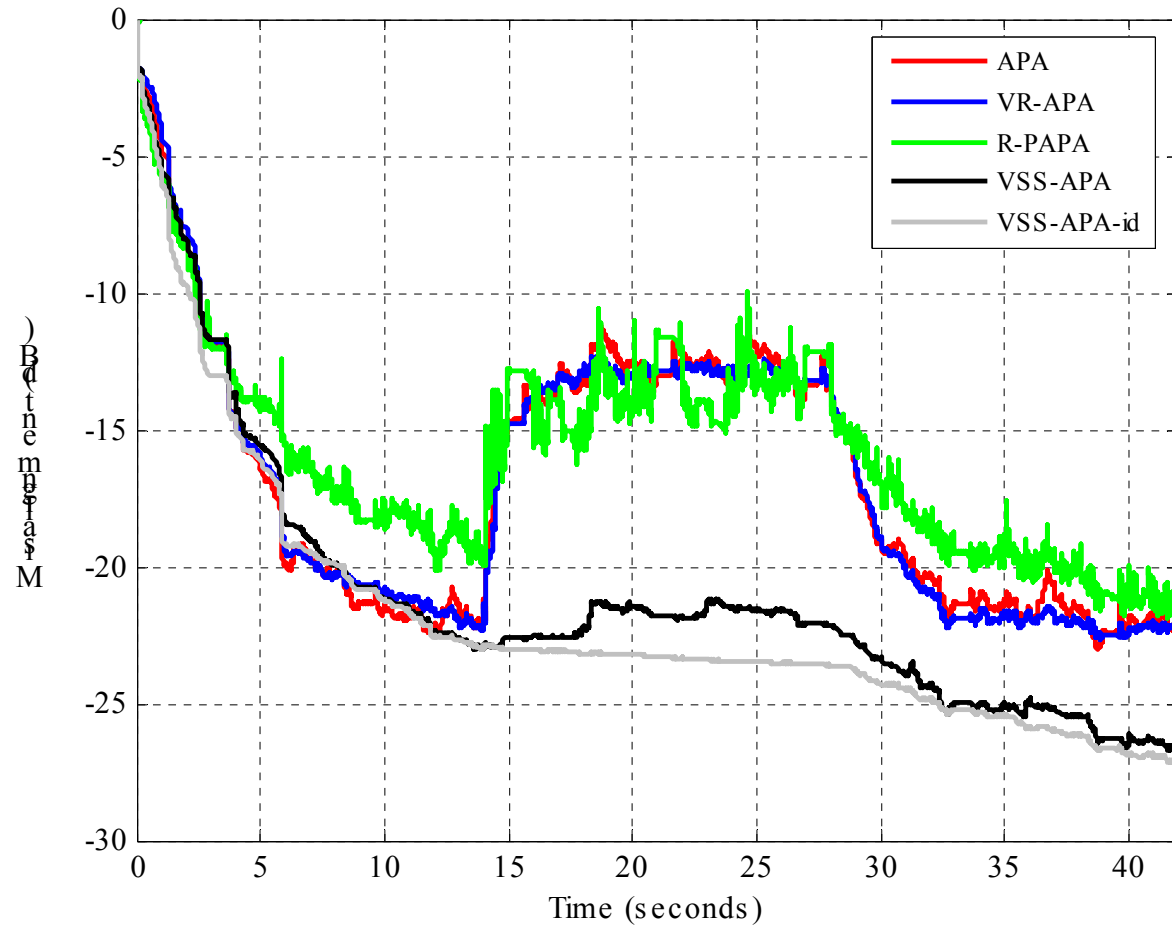
$p = 2$



$p = 8$

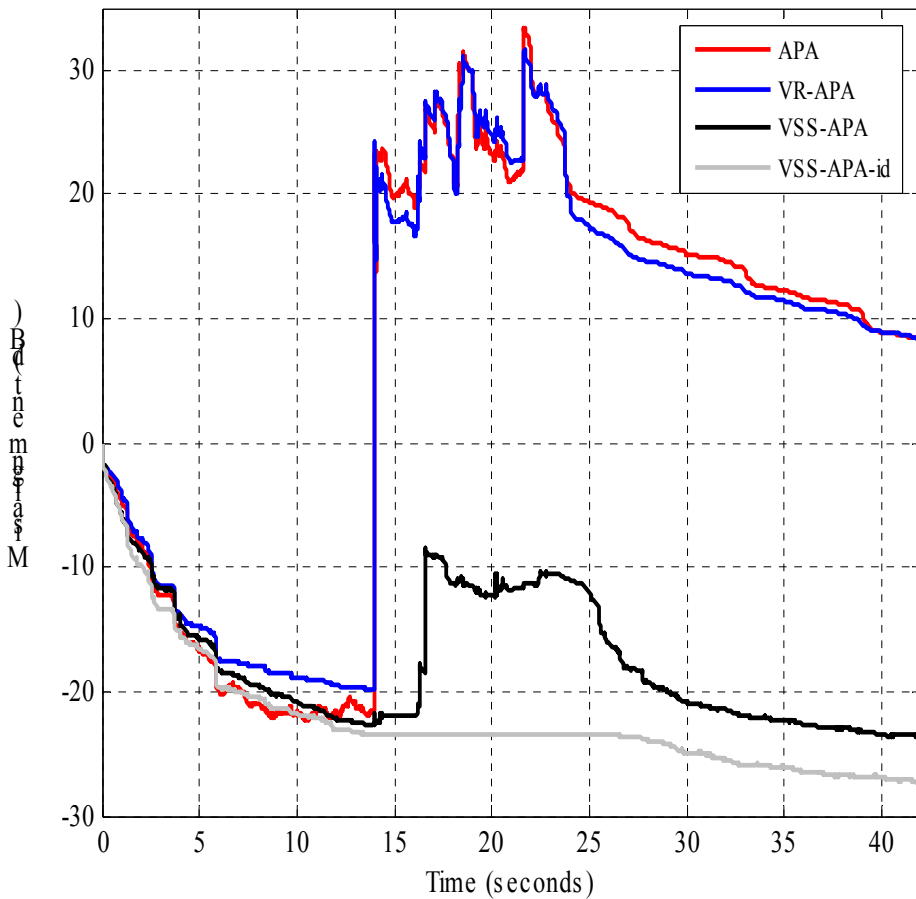


- single-talk scenario + background-noise increases (SNR  $\rightarrow$  20dB  $\downarrow$  10dB)

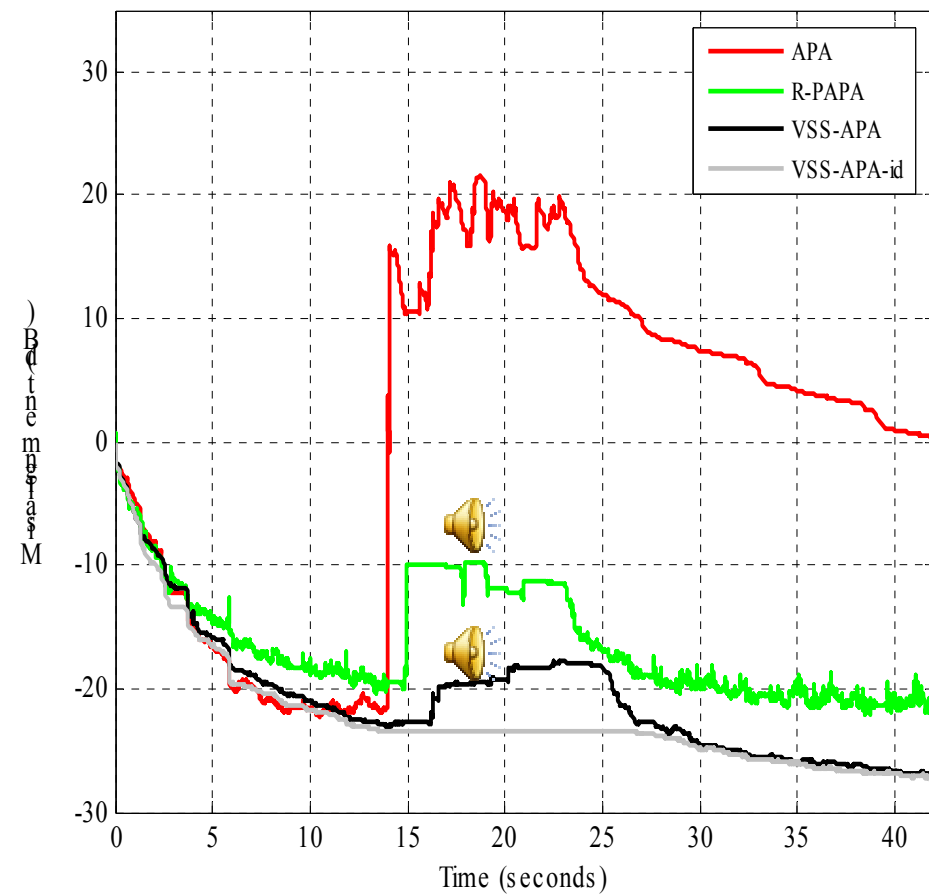


- double-talk scenario (near-end signal = background noise + near-end speech)

without DTD



with Geigel DTD



# Conclusions

- VSS-NLMS and VSS-APA were developed in the context of echo cancellation.
- they takes into account the existence of the near-end signal
- the VSS formula does not require any additional parameters from the acoustic environment (i.e., non-parametric)
- they are robust to near-end signal variations like the increase of the background noise or double-talk